Algorithmic Engagements contest

Presentation of solutions

A. Interesting Paths

Fastest solution: **UTokyo: Time Manipulators (0:12)**

A. Interesting Paths

Problem statement

Given is a DAG with n vertices and m edges.
What is the longest possible sequence of paths in which each path:

- starts in the source (vertex 1) and finishes in the sink (vertex n)
- contains at least one edge not contained in any of the previous paths

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- Let N be the number of vertices reachable both from the source and the sink (in the later case in the reversed graph).
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- The answer is $M - N + 2$.

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Complexity

$\mathcal{O}(n + m)$

B. Roars III

Fastest solution: **none :(**

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Given is a tree which in some of its vertices contain tokens. For each of its vertices independently we should assume that it is a root and solve the following problem:
In one move we can choose a vertex (different that the root) which contains a token and move this token one edge towards the root. We can do it only if the target vertex doesn't contain a token. We have to calculate the maximum possible number of moves.

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We can treat such sequence of moves as moving the deepest token directly to the root of the subtree.
We can find the deepest token in the subtree in time $\mathcal{O}(\log(n))$ using segment tree.

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- What if we calculated the answer for some vertex v and we want to calculate it for its neighbor u? – Only two moves can be different!
- Let's rollback these two moves (firstly rollback bringing the token to v and then to u) and then bring tokens to v and to u from correct subtrees.

Complexity

If we maintain a segment tree over the tree we can perform each operation in time $\mathcal{O}(\log(n))$ which gives the final complexity $\mathcal{O}(n\log(n))$.

C. Radars

Fastest solution: **UTokyo: Time Manipulators (0:07)**

C. Radars

Problem statement

Given is a square board $n \times n$. For each of its cells we know the cost of building a radar in it which will cover a square with side n centered in this cell. What is the minimal cost to cover the whole board?

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Conclusion

We can focus only on covering the corners.

Algorithm

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To consider a cell with a mask of corners m and cost x, we need to relax the dynamic programming – for each i we execute $DP[i|m] = \min(DP[i|m], DP[i] + x)$.

Complexity

Linear in the size of the board – $\mathcal{O}(n^2)$.

D. Xor Partitions

Fastest solution: **Harbour.Space: P+P+P (0:08)**

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Given is a sequence of integers $a_1, a_2, \ldots, a_n$.
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The value of a partition of the sequence into intervals is the product of the values of each interval.
Calculate the sum of the values of all the partitions of a.

D. Xor Partitions

Slow solution

- $dp[i]$ - sum of the values of all partitions of $a_1, a_2, \ldots, a_i$

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- Iterate over the length of the last interval in the partition.
- The sum of the values of such partitons – the value of the last segment multiplied by sum of the values of all partitions of the prefix.
- $$dp[i] = \sum_{j=0}^{i-1} dp[j] \cdot XOR(a_{j+1}, a_{j+2}, \ldots, a_i)$$

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Complexity $\mathcal{O}(n^2)$ – too slow.

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- $$dp[i] = \sum_{b} \sum_{j=0}^{i-1} dp[j] \cdot 2^b \cdot [\text{b is set in } XOR(a_{j+1}, a_{j+2}, \ldots, a_i)]$$

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- Set $pref_i = XOR(a_1, a_2, \ldots, a_i)$
- $$dp[i] = \sum_{b} \sum_{j=0}^{i-1} dp[j] \cdot 2^b \cdot [\text{state of b is different in } pref_i \text{ and in } pref_j]$$

D. Xor Partitions

Algorithm

We can calculate $DP_2[i][b][2]$ – the sum of $DP[j]$ with $j \leq i$ such that the bit b is set or not in $pref_i$. It's easy to update it and calculate DP with it.

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We can only remember the last layer of DP_2.

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Complexity

$\mathcal{O}(n \cdot \log(max(a_i)))$

E. Pattern Search II

Fastest solution: **UTokyo: Time Manipulators (1:21)**

E. Pattern Search II

Problem statement

Given is a string t over binary alphabet. We have to choose an equal to it subsequence of the infinite Fibonacci word, so that the distance between the first and the last chosen position is minimal.

Infinite word

Each 3 consecutive characters in Fibonacci word contain both letters, so the result won't exceed $3n$.

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We don't have to look for the optimal subseqnece too far.

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Speed up

We need to be able to answer the following queries: if we'd want to match to S_k the characters of t starting from the i-th one, how many of them would we match?

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Answers for all such queries can be easily calculated – if we denote the answer for the above question by $DP[i][k]$, then
$DP[i][k] = DP[i][k-1] + DP[i + DP[i][k-1]][k-2]$ holds.

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Complexity

Dynamic programming and looking for all the subsequences take time $\mathcal{O}(n \cdot \log(n))$ each.

F. Waterfall Matrix

Fastest solution: **Add Train Team (1:41)**

F. Waterfall Matrix

Problem statement

We want to create a matrix $n \times n$ in which the values in all columns and rows are nonincreasing. For some subset of its cell we are told what should be in them. For each of these cells the penalty is the absolute difference between the required value and the value in our matrix. We have to minimize the sum of penalties.

F. Waterfall Matrix

Preparation

We can move given cells without changing the answer, so that all of them are in different rows and columns.

F. Waterfall Matrix

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Sweepline

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- We can use multiset and store the places in which the result increases.

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Border recovery

We can reverse this process and recover the optimal border – this will tell us which cells should be $\leq x$ and which should be $> x$.

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Key observation

Optimal border for x will be below and to the right of the optimal border of $x + 1$

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Optimal border for x will be below and to the right of the optimal border of $x + 1$ – we can find them independently and sum the results!
It would give us correct algorithm working in time $\mathcal{O}(n^2 \log(n))$.

F. Waterfall Matrix

Divide and conquer!

For any x we can check which cells should be greater than x and which shouldn't.

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We can run the algorithm for the "middle" value of x and split recursively on two sides passing each cell only to one side.

F. Waterfall Matrix

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Complexity

If we use a data structure (such as multiset) which works in $\mathcal{O}(\log(n))$ per operation we will end up with complexity $\mathcal{O}(n \log^2(n))$.

G. Puzzle II

Fastest solution: **Add Train Team (2:04)**

G. Puzzle II

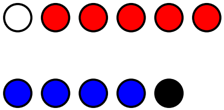
Problem statement

Given two binary sequences of length n and a number k. In one move we can choose a cyclic segment of length k from the first sequence and a cyclic segment of the same length from the second sequence and swap them. We need to make both sequences monochromatic in at most n moves.

What to do?

What organized moves can we do?

G. Puzzle II



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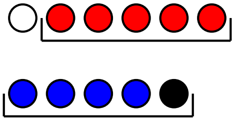

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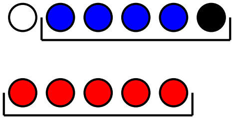

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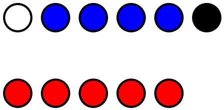

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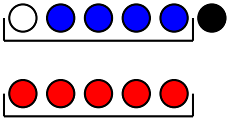

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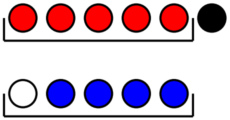

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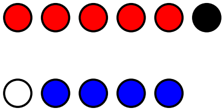

What to do?

In two moves we can move an element from the first sequence to the second and one from the second to the first.

G. Puzzle II

What to do?

In two moves we can move an element from the first sequence to the second and one from the second to the first. As we can choose which sequence will be white we can do at most $\frac{n}{2}$ such operations, resulting in $\leq n$ moves.

G. Puzzle II

Should we start?

An experienced eye should spot a solution which uses some BST and would result in a $\mathcal{O}(n \cdot \log(n))$ complexity...

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Should we start?

An experienced eye should spot a solution which uses some BST and would result in a $\mathcal{O}(n \cdot \log(n))$ complexity...However it might be a good idea to look for something simpler and faster.

Clever way

Let's set a sliding window of size $k + 1$ on first elements of the first sequence.

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As long as we are not happy with the first element of the first window we can slide it to the right – and the second window to the left.
We'll move both windows at most $\mathcal{O}(n)$ times.

G. Puzzle II

Complexity

$\mathcal{O}(n \cdot \log(n))$ with a BST of your choice or $\mathcal{O}(n)$ with some tricks.

H. Weather Forecast

Fastest solution: **LNU: LNU Stallions (0:17)**

H. Weather Forecast

Problem statement

We are given a sequence of integers and a number k. We need to find a partition of this sequence into k intervals which maximizes the minimum mean over all intervals.

Key observation

If we can have all means $\geq x$, we surely can have all means $\geq y$ if $x \geq y$.

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Conclusion

Binary search to find the answer.

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Solution

Longest increasing sequence.

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Solution

Longest increasing sequence.

Complexity

$\mathcal{O}(n \cdot \log(n) \cdot \log(precision))$

I. Mercenaries

Fastest solution: **UTokyo: Time Manipulators (4:10)**

I. Mercenaries

Problem statement

We are given a straight road on which we can move only to the right. On the road there are n cities and the mercenary living in the i-th city is parametrized by pair (s_i, m_i). Between each two neighboring cities there is a shop which allows to buy one item in it and each item will add some values to both statistics of the mercenary. When mercenary moves from one city to another (he can move only to the right) he can buy one item in each shop and their bonuses will accumulate. We have to consider monster attack scenarios – a monster can attack some city and this monster is parametrized by three values – A, B and C. A mercenary can defeat the monster if he can get to it having statistics (S, M) so that $A \cdot S + B \cdot M \geq C$. We have to find the rightmost mercenary which could defeat each monster.

Is this geometry?

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I. Mercenaries

Is this geometry?

Let's interpret mercenaries and items as vectors in the first quarter of a coordinate plane – defeating the monster means being in a given halfplane.

Is this convex hulls?

To check if a monster can be defeated we need to consider only upper-right convex hull of the statistics of the mercenaries that can get to this monster.

Organized approach

Let's build a segment tree on the sequence of cities.

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Organized approach

Let's build a segment tree on the sequence of cities.
In each base segment let's calculate a convex hull of possible bonuses that we can get if we pass through this segment.
A convex hull for a segment is a Minkowski sum of convex hulls for its two subsegments – we can merge them in linear time.

Organized approach part two

For each base segment let's also calculate a convex hull of possible statistics of mercenaries which start in this interval in the moment they leave it.

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Such a mercenary can start in the right subsegment or in the left one and strengthen himself with items from the right interval

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Such a mercenary can start in the right subsegment or in the left one and strengthen himself with items from the right interval (again Minkowski sum).
Convex hull of a set of points also can be calculated in linear time.

Attack scenario

When a monster attacks let's split the prefix into base segments and consider them from right to left.

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If yes, we go deeper in the tree.
If no, we have to consider the items that the mercery can buy. Their maximum possible impact on the monster (information how much should we decrease the C parameter) also can be found with binary search.

Optimization

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Mentioned optimization will allow us to multiply the size of the input by only one logarithm – the height of the segment tree and the need of sorting the monsters and items by angle.

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Complexity

Mentioned optimization will allow us to multiply the size of the input by only one logarithm – the height of the segment tree and the need of sorting the monsters and items by angle. We'll end up with comlexity $\mathcal{O}((n + \sum_i r_i + q) \cdot \log(n + \sum_i r_i))$.

J. Polygon II

Fastest solution: **Harbour.Space: P+P+P (3:24)**

J. Polygon II

Problem statement

Random variables $X_1, X_2, \ldots, X_n$, where $X_i = U(0, 2^{a_i})$. Find the probability that we can construct a nondegenerate polygon with sides of lengths X_i.

J. Polygon II

Triangle inequality

Bad if and only if for some i

$$X_i \geq \sum_{j \neq i} X_j.$$

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Answer

$$1 - \sum_i P(X_i \geq \sum_{j \neq i} X_j)$$

– bad events are disjoint.

Helpful lemma

$$P(X_i \geq \sum_{j \neq i} X_j) = P(2^{a_i} \geq \sum_{j} X_j)$$

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J. Polygon II

Main idea

Let Y_i be a random variable with only two possible values:

$$P(Y_i = 0) = \frac{1}{2}, P(Y_i = 2^i) = \frac{1}{2}$$

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Bits decomposition

$$X_i = U(0,1) + Y_0 + Y_1 + \ldots + Y_{a_i - 1}$$

Dynamic programming

$DP[i][j]$ – probability, that we carry j bits (of value 2^i) after deciding on all $U(0,1)$, Y_0, Y_1, … Y_{i-1}.

J. Polygon II

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Transitions

k_i – number of variables of type Y_i

$$DP[i+1][j] = \sum_{l=0}^{k_i} (DP[i][2j-l] + DP[i][2j-l+1]) \frac{\binom{k_i}{l}}{2^{k_i}}$$

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$\sum_{i=0}^{j} DP[0][i] =$ volume of an n-dimentional polyhedron $\sum x_i < j$ and $0 \leq x_i \leq 1$.

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Initialization – only $U(0,1)$

$\sum_{i=0}^{j} DP[0][i] =$ volume of an n-dimentional polyhedron $\sum x_i < j$ and $0 \leq x_i \leq 1$.
Inclusion-exclusion principle on how many $x_i \leq 1$ are not met.

J. Polygon II

Complexity

$\mathcal{O}(max(a_i) \cdot n^2)$

J. Polygon II

Complexity

$\mathcal{O}(max(a_i) \cdot n^2)$ or $\mathcal{O}(max(a_i) \cdot n \cdot \log(n))$ with FFT.

K. Power Divisions

Fastest solution: **Add Train Team (1:18)**

K. Power Divisions

Problem statement

Given is a sequence $b_1, b_2, \ldots b_n$ of form $2^{a_1}, 2^{a_2}, \ldots, 2^{a_n}$.
An interval $[l, r]$ is good $\iff b_l + b_{l+1} + \ldots + b_r = 2^k$ (for $k \in \mathbb{N}$)
Calculate the number of partitions of the sequence into good intervals (modulo prime number).

Workflow

K. Power Divisions

Workflow

- Find all good intervals

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- Find all good intervals – divide&conquer.

K. Power Divisions

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- Find all good intervals – divide&conquer.
- Count all good partitions

K. Power Divisions

Workflow

- Find all good intervals – divide&conquer.
- Count all good partitions – dynamic programming.

Representation of sum S and increasing it by b_i

K. Power Divisions

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S	101110
b_i	000100
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Time – amortized $\mathcal{O}(1)$ (potential – number of set bits).

K. Power Divisions

Constants

P – big prime number, for example $2^{61} - 1$.

$c_0, c_1, \ldots, c_{10^6+20}$ – random coefficients.

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$h(S) = \sum_i b_i \cdot c_i$ modulo P.

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Probability of a collision

$S_1 \neq S_2 \Rightarrow P(h(S_1) = h(S_2)) = \frac{1}{P}$

All good intervals – divide&conquer

K. Power Divisions

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- split the interval into L and R

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K. Power Divisions

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For $pref_R$ there is only one possible value of suf_L.

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2^k	1000000
suf_L	0010010

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Calculating $h(suf_L)$

a – rightmost set bit in $pref_R$.
b – leftmost set bit in $pref_R$.

K. Power Divisions

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b – leftmost set bit in $pref_R$.

$$h(suf_L) + h(pref_R) = c_a + \sum_{i=a}^{b} c_i.$$

K. Power Divisions

Intervals $suf_L + pref_R = 2^k$ – algorithm

- Memorize $h(suf_L)$ – (hash)map from $h(suf_L)$ into L.

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K. Power Divisions

Intervals $suf_L + pref_R = 2^k$ – algorithm

- Memorize $h(suf_L)$ – (hash)map from $h(suf_L)$ into L.
- Iterate over $pref_R$, keep frack of S, $h(S)$, a and b.
- Check if good suf_L exists.

K. Power Divisions

Intervals $suf_L + pref_R = 2^k$ – algorithm

- Memorize $h(suf_L)$ – (hash)map from $h(suf_L)$ into L.
- Iterate over $pref_R$, keep frack of S, $h(S)$, a and b.
- Check if good suf_L exists.

Probability of a collision

$\sum_{suf_L} \sum_{pref_R} P(h(suf_L) = h(2^k - pref_R)) \leq \frac{n^2}{P}$

Complexity

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- Dynamic programming: $\mathcal{O}(\text{number of good intervals}) = \mathcal{O}(n \log n)$.

L. Chords

Fastest solution: **Harbour.Space: P+P+P (2:36)**

L. Chords

Problem statement

$2n$ points on a circle were randomly paired creating n chords. We need to find the biggest subset of chords such that no two of them intersect.

Simpler look

We can cut the circle in any position

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If in point r some chord ends, its begining is on the point $left_r$ and $left_r \geq \ell$ holds, we need to consider $DP[\ell][left_r - 1] + 1 + dp[left_r + 1][r - 1]$.

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Described dynamic programming calculates correct answer in time $\mathcal{O}(n^2)$ and returns it in $DP[1][2n]$.

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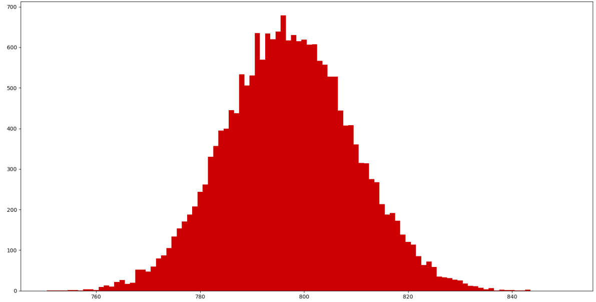

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Complexity

We end up with time and memory complexity $\mathcal{O}(n \cdot ans)$.

M. Balance of Permutation

Fastest solution: **Harbour.Space: P+P+P (1:31)**

M. Balance of Permutation

Problem statement

A balance of a permutation p is defined as the sum of $|p_i - i|$. We have to find k-th lexicographically smallest n-element permutation with balance equal to b.

M. Balance of Permutation

Simple look

A permutation is as assignment of values to positions – let's imagine n red numbers (the positions) and n blue numbers (the values) and sort them as numbers.

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Knowing the sum of right numbers, we also know the sum of left numbers, so we know the balance.
Such dynamic programming has $\mathcal{O}(n^4)$ states and we calculate each of them in constant time.

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The final complexity will be $\mathcal{O}(n^6)$.